

Novel Properties of Massive Higher Spin Fields

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I outline a series of results obtained in collaboration with A. Waldron on the properties of massive higher ($s > 1$) spin fields in cosmological, constant curvature, backgrounds and the resulting unexpected qualitative effects on their degrees of freedom and unitarity properties. The dimensional parameter Λ extends the flat space m -line to a (m^2, Λ) “phase” plane in which these novel phenomena unfold. In this light, I discuss a possible partial resurrection of deSitter supergravity. I will also exhibit the well-known causality problems of coupling these systems to gravity and, for complex fields, to electromagnetism, systematizing some of the occasionally misunderstood obstacles to interactions, particularly for $s = 3/2$ and 2.

1. Introduction

It is a particular pleasure to report here on work with Andrew Waldron on properties of higher spin fields, because these systems have a longtime and broad following here in Brazil. Although I will not assume universal familiarity with the background, space will not permit me to delve into any details either. For these, I refer you to our papers [1] where you will also find references to guide you through the enormous and often contradictory earlier literature. A more detailed review article should also appear in due course.

Observationally, we seem to live in a universe whose vacuum state is the deSitter (dS) geometry of constant positive, rather than flat, curvature. At the same time we know that string theory, when expanded in terms of local actions at low energy, gives rise to towers of massive higher spin fields beyond the currently observed world of the standard model, with spin 0, $\frac{1}{2}$ and 1. These broad facts immediately force us to go beyond flat space, where representations of the Poincaré group simply fall into a massless/massive dichotomy, the former having 2, and the latter have $2s+1$, degrees of freedom (DoF) in $D=4$. This is not entirely unknown territory – it was initially explored by Dirac, in the 1930s and the so-called cosmological extensions of $N=1$

SUGRA also seemingly required “massive” and “gauge” spin $3/2$ particles in Anti-deSitter (AdS, $\Lambda < 0$) gravity. The first “modern” results [2] concerned (linear, in no way “graviton-like”) massive spin 2 fields, where the unusual properties we will develop here are already visible: At “tuned” values of the dimensionless ratio m^2/Λ , novel local gauge invariances appear and remove the helicity 0 component of the original model’s helicities $(\pm 2, \pm 1, 0)$. A systematic study [3] of the corresponding 2-point functions then revealed the presence of a forbidden, negative norm, region of this m^2/Λ ratio. This turns out to be the tip of a major iceberg: “partial masslessness”, with associated novel gauge invariances and unitarily forbidden regions of the (m^2, Λ) plane occurs for all fermions and bosons with spin $s > 1$. Important consequences include null propagation at the critical values, as well as energy stability and unitarity properties of the corresponding quanta. These may even affect the old question whether SUGRA is tenable in dS, despite the fact that the spin $3/2$ particle’s mass parameter is proportional to $\sqrt{-\Lambda}$.

The second set of questions – consistency difficulties of coupling massive spin >1 fields to gravity (not just in (A)dS!), and if complex, to electromagnetism, is a very old and actually well-understood topic particularly for the key, $s = 3/2$ and $s = 2$, cases. Our systematization of it is

mentioned here because there are still occasional misunderstandings (even in Brazil) of the established results. We use the notion of characteristics to show that acausality can be seen to arise as the onset of spacelike propagation at strong enough external magnetic fields, for example, in addition to the other ills, namely loss of correct DoF count, because essential free-field constraints can be negated by the coupling.

2. de Sitter Space

We begin with a brief summary of our geometries: dS_{n+1} spacetime, the coset $SO(n+1,1)/SO(n,1)$, geometrically described by the one-sheeted hyperboloid

$$\begin{aligned} Z^M Z_M &= -(Z^0)^2 + \sum_{i=1}^n (Z^i)^2 + (Z^{n+1})^2 = 1, \\ M &= 0 \dots n+1, \end{aligned} \quad (1)$$

embedded in $\mathbb{R}^{(n+1,1)}$. The cosmological constant Λ is positive in dS space and usually enters the equation (1) as n/Λ ($=1$ in our units) on the right hand side. Lack of space precludes details of the analogous AdS picture, where $SO(n,2)$ is the isometry group.

The dS group $SO(n+1,1)$ acts naturally on $\mathbb{R}^{(n+1,1)}$ with generators

$$M_{MN} = i(Z_M \partial_N - Z_N \partial_M) \quad (2)$$

obeying the Lie algebra

$$\begin{aligned} [M_{MN}, M_{RS}] &= i\eta_{NR} M_{MS} - i\eta_{NS} M_{MR} \\ &+ i\eta_{MS} M_{NR} - i\eta_{MR} M_{NS}. \end{aligned} \quad (3)$$

Using a coordinate system in which spatial sections are flat, one can rewrite this algebra in terms of

$$\begin{aligned} P_i &\equiv M_{i0} + M_{n+1,i}, \quad D \equiv M_{n+1,0}, \\ K_i &\equiv M_{i0} - M_{n+1,i}, \quad M_{ij} \end{aligned} \quad (4)$$

The latter satisfy the $so(n)$ angular momentum Lie algebra, so identifying the remaining generators P_i as momenta, D as dilations and K_i as conformal boosts, their algebra generates the Euclidean conformal group in n dimensions, with $i, j = 1 \dots n$.

In terms of the embedding coordinates

$$Z^M = (gX)^M = \left(\sinh t + \frac{1}{2} e^t \vec{x}^2, e^t \vec{x}, \cosh t - \frac{1}{2} e^t \vec{x}^2 \right), \quad (5)$$

the interval becomes

$$\begin{aligned} ds^2 &= dZ^M \eta_{MN} dZ^M \\ &= -dt^2 + e^{2t} d\vec{x}^2 \equiv dx^\mu g_{\mu\nu} dx^\nu. \end{aligned} \quad (6)$$

While the parameterization of the coset only spans one half, $Z^{n+1} > Z^0$, of the hyperboloid, the physical region within the intrinsic horizon

$$0 > -1 + e^{2t} \vec{x}^2 = \xi^\mu \xi_\mu, \quad \xi^\mu = (-1, x^i), \quad (7)$$

is covered by this coordinate patch.

3. Arbitrary Spin Bosons

Let us now insert matter in our dS (we have only space for bosons). A massive spin s field can be described by a completely symmetric tensor $\varphi_{\mu_1 \dots \mu_s}$ subject to the field equation and constraints

$$\begin{aligned} (D_\mu D^\mu - 2n + 4 + (n-5)s + s^2 - m^2) \varphi_{\mu_1 \dots \mu_s} \\ = 0 = D \cdot \varphi_{\mu_2 \dots \mu_s} = \varphi^\rho{}_{\rho \mu_3 \dots \mu_s}. \end{aligned} \quad (8)$$

For generic values of the mass m , $\varphi_{\mu_1 \dots \mu_s}$ describes the

$$\mu(n, s) \equiv \frac{(n+2s-2)(n+s-3)!}{s!(n-2)!} \quad (9)$$

degrees of freedom of a spin s symmetric field in $n+1$ dimensions. The mass parameter is defined so that the theory is strictly massless for $m^2 = 0$ with a gauge invariance

$$\varphi_{\mu_1 \dots \mu_s} = \partial_{(\mu_1} \xi_{\mu_2 \dots \mu_s)}, \quad (10)$$

(subject to $\xi^\rho{}_{\rho \mu_3 \dots \mu_s} = 0$). The degree of freedom count is then

$$\mu(n, s) - \mu(n, s-1) = \frac{(n+2s-3)(n+s-4)!}{s!(n-3)!}. \quad (11)$$

Actions may be written down for these free theories, both massive and massless.

The physical polarizations of a strictly massless field satisfy

$$\partial \cdot V_{(s-1)} \equiv \partial^i V_{i i_2 \dots i_s} = 0, \quad (12)$$

thanks to the gauge invariance (10) which projects out all but the maximal helicity s excitations. A field obeying (12) has the correct degree of freedom count, as given in (11), for a strictly massless field.

For partially massless fields, gauge invariances of the form

$$\delta\varphi_{\mu_1\dots\mu_s} = \mathcal{D}_{(\mu_1\dots\mu_t}\varphi_{\mu_{t+1}\dots\mu_s)} + \dots \quad (13)$$

imply that the requirement (12) is relaxed and replaced by

$$\partial^{i_1}\dots\partial^{i_t}V_{i_1\dots i_s} = 0, \quad (t \leq s). \quad (14)$$

We call such a field “partially massless of depth t ”. This amounts to projecting out all helicities save $(s, \dots, t+1)$ and gives $\mu(n, s) - \mu(n, s-t)$ degrees of freedom.

The subalgebra of translations, dilations and rotations leaves the condition (14) invariant. However, conformal boosts do not, unless one tunes the conformal weights Δ_s appropriately. To obtain these tunings we study

$$\partial^{i_1}\dots\partial^{i_t}K_iV_{i_1\dots i_s} = 0, \quad (15)$$

for depth t partially massless polarizations $V_{(s)}$ subject to (14). It is a simple combinatorics problem to compute the (unique) value of Δ_s as a function of the depth t such that the condition (15) holds. We state the result below. The main idea is conveyed by the simplest non-trivial example, spin 2.

For a spin 2 field $V_{ij} - V_{ji} = 0 = V_i^i$, the conformal boost acts as

$$\begin{aligned} iK_iV_{jk} &= i(-2y_iD + \vec{y}^2P_i)V_{jk} \\ &+ 4y_{(j}V_{k)i} - \delta_{i(j}y_lV_{k)l}. \end{aligned} \quad (16)$$

The field V_{ij} is strictly massless whenever

$$\partial^iV_{ij} = 0, \quad (17)$$

so we test whether this condition is respected by conformal boosts by computing

$$\partial^kK_iV_{jk} = 2i(\Delta_s - n)V_{ij}. \quad (18)$$

Here we have relied on the divergence constraint (17). Hence we find the strictly massless tuning

$$\Delta_s = n. \quad (19)$$

Using this relation as well as $s = 2$ gives $m^2 = 0$, the correct tuning for a strictly massless spin 2 boson.

To study partially massless spin 2, we replace the single divergence condition (17) by the double divergence one

$$\partial^{ij}V_{ij} = 0. \quad (20)$$

Now we must compute

$$\partial^j\partial^kK_iV_{jk} = 4i(\Delta_s - n + 1)\partial_jV_{ij} \quad (21)$$

where we used (20) but *not* (17). Therefore we obtain the partially massless spin 2 tuning

$$\Delta_s = n - 1. \quad (22)$$

It is also clear that the partially massless representation is irreducible. One might have thought it to be a direct sum of spin 2 and spin 1 strictly massless representations. However, since the tunings (19) and (22) differ, the strictly massless spin 2 field components satisfying (17) mix with the remaining ones when $\Delta_s \neq n$.

We calculate the tunings for arbitrary spin in the same way by imposing (14) and computing

$$\begin{aligned} \partial^{i_1}\dots\partial^{i_t}K_iV_{i_1\dots i_s} \\ = 2it(\Delta_s - n - s + t + 1)\partial^{i_2}\dots\partial^{i_t}V_{ii_2\dots i_s}. \end{aligned} \quad (23)$$

The tunings are therefore

$$\Delta_s = n + s - t - 1. \quad (24)$$

Inserting the tuning condition (24) in the dS mass-conformal weight relation yields the mass conditions for depth t partial masslessness

$$m^2 = (t-1)(2s-3+n-t). \quad (25)$$

Note that for depth $t = 1$, *i.e.* strictly massless fields, the mass parameter $m^2 = 0$. When $n = 3$, the result agrees with requiring light cone propagation for all the helicities of partially massless fields. The above results may be summarized by Figure 1, where the full (m^2, Λ) phase plane for both bosons and fermions is displayed.

4. Spin 2

We give here a concrete example of the behavior of a model at critical tuned values of m^2/Λ ,

namely the (simplest) $s=2$ case in $D=4$. After some work, the action decomposes into a non-interacting sum of helicities, in terms of the standard flat space orthogonal decomposition of a symmetric tensor T_{ij} ,

$$I = I_{\pm 2}(T_{ij}^{TT}) + I_{\pm 1}(T_i^T) + I_0(T^t, T^t), \quad (26)$$

where the subscripts refer to the helicities covered. Let us begin with the helicity ± 2 part, where there is never a constraint:

$$I_{\pm 2} = p_{\pm 2} \dot{q}_{\pm 2} - \left[\frac{1}{2} p_{\pm 2}^2 + \frac{1}{2} q_{\pm 2} \left(-\nabla^2 + m^2 - \frac{9M^2}{4} \right) q_{\pm 2} \right]. \quad (27)$$

[Here and throughout, we omit all explicit integrals.] We will explain and meet again the effective mass $(m^2 - 9M^2/4)$ later, and at present just state that this action ensures stable, unitary propagation for *all* $m^2 \geq 0$. Therefore the helicity ± 2 modes propagate according to (28) for all models in the (m^2, Λ) half-plane; here $M^2 \equiv \Lambda/3$.

The helicity ± 1 action is identical to its ± 2 counterpart with one important difference: The field redefinition needed to reach this form is singular at $m^2 = 0$, as it should be, since I_{\pm} , disappears at $m^2 = 0$.

The physical helicity 0 state leads a more interesting life, as it can be (i) stable and unitary when $m^2 > 2M^2$, (ii) absent when $m^2 = 2M^2$ or (iii) unstable and nonunitary for $m^2 < 2M^2$. After a great deal of work the full helicity 0 action becomes

$$\begin{aligned} I_0 &= p_0 \dot{q}_0 - \left[-\frac{3\nu^2 m^2}{32M^2} (h^t)^2 - \frac{1}{2M} h^t \right. \\ &\quad \left(\nabla^2 [p_0 - Mq_0] + \frac{\nu^2 m^2}{4M} q_0 \right) \\ &\quad - \frac{2}{m^2} [p_0 - Mq_0] \nabla^2 [p_0 - Mq_0] \\ &\quad \left. + \frac{3}{2} [p_0 - Mq_0] \left(p_0 - \frac{m^2}{3M} q_0 \right) \right], \\ \nu^2 &\equiv (m^2 - 2M^2). \end{aligned} \quad (28)$$

The denominators M in this expression do not represent a genuine singularity, but arise from choosing to solve the constraint in terms of p^t . In contrast, the denominators m^2 are due to integrating out the shift N_i and are a reminder (as we

have seen already) of the strictly massless $m^2 = 0$ gauge theory. The key point is to notice that the coefficient of $(h^t)^2$ vanishes on the critical line $\nu^2 = 0$ (as well as at $m^2 = 0$, concordant with the previous remark). At criticality, the field h^t appears only linearly and is a Lagrange multiplier for a new constraint, whereas for $\nu^2 \neq 0$, we can integrate out h^t by its algebraic field equation and there are no further constraints. Let us deal with each of these cases in turn.

Consider the models with mass tuned to the cosmological constant via $m^2 = 2\Lambda/3$. As is clear from (28), the Lagrange multiplier h^t imposes the new constraint $p_0 - Mq_0 = 0$. Eliminating q_0 (say) and since $(p_0 \dot{p}_0)$ is a total derivative, the 0 helicity action (28) vanishes exactly, $I_0 = 0$. It is known [2] that the critical theory possesses a local scalar gauge invariance,

$$\delta h_{\mu\nu} = (D_{(\mu} D_{\nu)} + \frac{\Lambda}{3} \bar{g}_{\mu\nu}) \xi(x). \quad (29)$$

Thus, our result establishes that its effect is to remove the lowest helicity excitation. Therefore, the total action is

$$I_{\nu^2=0} = \sum_{\epsilon=(\pm 2, \pm 1)} \left\{ p_{\epsilon} \dot{q}_{\epsilon} - \left[\frac{1}{2} p_{\epsilon}^2 + \frac{1}{2} q_{\epsilon} \left(-\nabla^2 - \frac{M^2}{4} \right) q_{\epsilon} \right] \right\}. \quad (30)$$

[The effective mass $-M^2/4$ is the same one as in (27), evaluated at $\nu^2 = 0$.] These remaining helicity $(\pm 2, \pm 1)$ excitations are both unitary and, as we will show, stable.

We may now eliminate h^t by its algebraic field equation

$$h^t = -\frac{8M}{3\nu^2 m^2} \nabla^2 [p_0 - Mq_0] - \frac{2}{3} q_0, \quad (31)$$

and made a penultimate field redefinition/canonical transformation to obtain

$$\begin{aligned} I_0 &= p_0 \dot{q}_0 - \left[\frac{1}{2} \left[\frac{\nu^2 m^2}{12M^2} \right] q_0^2 + \frac{1}{2} \left[\frac{12M^2}{\nu^2 m^2} \right] p_0 \right. \\ &\quad \left. \left(-\nabla^2 + m^2 - \frac{9M^2}{4} \right) p_0 \right]. \end{aligned} \quad (32)$$

Before we present the final, complete, action, some important comments on its penultimate form (32) are needed:

- The sign of the parameter ν^2 controls the positivity of the Hamiltonian (and consequently the energy). Therefore we find that the (m^2, Λ) plane is divided into a stable region $m^2 \geq 2\Lambda/3$ and an unstable one $m^2 < 2\Lambda/3$. We have already dealt with the transition line $m^2 = 2\Lambda/3$.
- A final field redefinition,

$$p_0 \rightarrow -\frac{\nu m}{2\sqrt{3} M} q_0, \quad q_0 \rightarrow \frac{2\sqrt{3} M}{\nu m} p_0 \quad (33)$$

brings the helicity 0 action into the same form as its helicity $(\pm 2, \pm 1)$ counterparts (27), but this is only legal in the stable massive region $m^2 > 2\Lambda/3$.

- The apparent singularity at $M = 0$ is spurious and reflects our (arbitrary) choice of solution to the constraint.

The final action for massive spin 2 in the region $m^2 > 2\Lambda/3$ is

$$I_{\nu^2 > 0} = \sum_{\epsilon=(\pm 2, \pm 1, 0)} \left\{ p_\epsilon \dot{q}_\epsilon - \left[\frac{1}{2} p_\epsilon^2 + \frac{1}{2} q_\epsilon \left(-\nabla^2 + m^2 - \frac{9M^2}{4} \right) q_\epsilon \right] \right\}, \quad (34)$$

and describes $2s + 1 = 5$ stable, unitary, helicity $(\pm 2, \pm 1, 0)$ excitations.

We are now ready to demonstrate the stability of the model in the allowed region $m^2 \geq 2\Lambda/3$. The dS background possesses a Killing vector

$$\bar{\xi}^\mu = (-1, Mx^i), \quad \bar{\xi}^2 = -1 + (fMx^i)^2, \quad (35)$$

timelike within the intrinsic horizon $(fMx^i)^2 = 1$. Therefore, in this region of spacetime, it is possible to define a conserved energy whose positivity guarantees the stability of the model.

Let us consider helicity ϵ (omitting 0 at criticality) described by the conjugate pair (p_ϵ, q_ϵ) , whose time evolution is generated by the Hamiltonian

$$H_\epsilon = \frac{1}{2} p_\epsilon^2 + \frac{1}{2} q_\epsilon \left(-\nabla^2 + m^2 - \frac{9M^2}{4} \right) q_\epsilon. \quad (36)$$

However, H_ϵ is not conserved, thanks to the explicit time dependence $f^{-2}(t)$ in ∇^2 , which was to be expected since it generates time evolution $\frac{d}{dt}$ rather than along the Killing direction $\bar{\xi}^\mu \partial_\mu$. Instead, the conserved energy is defined in terms of the stress tensor

$$E_\epsilon = T_{\epsilon\mu}^0 \bar{\xi}^\mu = -T_{\epsilon 0}^0 + M x^i T_{\epsilon i}^0. \quad (37)$$

The momentum density $T_{\epsilon i}^0$ will be defined below and $-T_{\epsilon 0}^0 = H_\epsilon$. For gravity, the momentum density T_i^0 is the quadratic part of the coefficient of N_i , and a similar result holds here. Keeping track of our field redefinitions, we find (modulo irrelevant superpotentials)

$$T_{\epsilon i}^0 = -p_\epsilon \partial_i q_\epsilon + \frac{1}{2} \partial_i (p_\epsilon q_\epsilon). \quad (38)$$

It is not difficult to verify that the energy function

$$E_\epsilon = H_\epsilon - M x^i p_\epsilon \partial_i q_\epsilon - \frac{3}{2} M p_\epsilon q_\epsilon, \quad (39)$$

is indeed conserved, $\dot{E} = 0$.

Finally we come to positivity. Here we need only a simple extension of a method used in analyzing energy in dS. Rewriting E as

$$\begin{aligned} E &= \frac{1}{2} (\hat{x}^i \tilde{p}_\epsilon)^2 + \frac{1}{2} (f^{-1} \partial_i q_\epsilon)^2 \\ &\quad - fM|x| (\hat{x}^i \tilde{p}_\epsilon) (f^{-1} \partial_i q_\epsilon) + \frac{1}{2} m^2 q_\epsilon^2, \\ \tilde{p}_\epsilon &\equiv p_\epsilon - \frac{3M}{2} q_\epsilon, \quad x^i \equiv |x| \hat{x}^i, \end{aligned} \quad (40)$$

the first three terms are positive by the triangle equality within the intrinsic dS horizon

$$fM|x| < 1, \quad (41)$$

and the fourth, mass term is manifestly positive¹. This concludes our stability proof.

¹The Killing energy of a massive scalar in dS also takes the form (40) and is therefore stable for $m^2 \geq 0$. [In this non-gauge example, there is no analog to the spin 2 instabilities at negative values of m^2 or ν^2 whose vanishing is associated with gauge invariances.] Scalars in AdS actually enjoy a somewhat wider stability range, extending to negative values of m^2 due to a shift in the spectrum of the AdS 3-Laplacian. This broadening is unlikely for spin 2, since its stability is controlled entirely by the above gauge coefficients.

The instability of the region $m^2 < 2\Lambda/3$ is also manifest: Consider helicity $\epsilon = 0$. Recall that once $\nu^2 < 0$, we cannot make the rescalings with factors ν and ν^{-1} in the final field redefinition (33). This does not prevent us from constructing a conserved energy with a “triangle” form (40), but the *caveat* is that p_0^2 carries a factor ν^2 and likewise q_0^2 a factor ν^{-2} . Therefore the energy is negative and the theory is unstable in this region, entirely due to helicity 0.

5. Inconsistencies of massive charged gravitating higher spins

As a final topic, we rapidly review the problems of higher spin couplings to gauge and gravity fields, and a shorter way to obtain them. Localized, massive $s > 1$ particles have never been observed, in agreement with a large (if somewhat confusing) higher spin lore that they cannot be made to interact consistently even with gravity or electromagnetism. Here we combine two earlier lines of analysis into a systematic study of higher spins coupled to Einstein–Maxwell fields.

(i) Massive higher spins propagate consistently in constant curvature backgrounds for a range of parameters (m^2, Λ) centered around the Minkowski line $(m^2, 0)$. (ii) The original unitarity (or equivalently causality) difficulties of massive $s = 3/2$ persist in pure E/M backgrounds, even including all possible non-minimal couplings.

Our first new result is that the onset of the unitarity/causality difficulty for massive $s = 3/2$ in pure E/M backgrounds can be traced to a novel gauge invariance of the timelike component of the Rarita–Schwinger equation at E/M field strengths tuned to the mass. Although the full system is not invariant, a consequence of this invariance is signal propagation with lightlike characteristics. Beyond this tuned point, *i.e.*, for large enough magnetic field $\bar{B}^2 > (\frac{3m^2}{2e})^2$ (or better, small/large enough mass/charge), the system is neither causal nor unitary. This is an old result [4] but its rederivation in terms of a gauge invariance is edifying. Our second $s = 3/2$ result is that in *dynamical* Maxwell–Einstein backgrounds, causality can be maintained for any choice of E/M field, for certain values of the mass.

The first instance in which there is no underlying protection, in contrast to that provided by SUGRA for $s = 3/2$ is $s = 2$ is tensor theory: when charged it bears little resemblance to its one conceivable relative, Einstein gravity. Charged massive $s = 2$ preserves the correct DoF only for gyromagnetic ratio $g = 1/2$, but even this theory suffers from the usual causality difficulties. Furthermore, it has no good DoF coupling to general gravitational backgrounds, so there is no analog of the causality bounds found for $s = 3/2$.

We examine the leading discontinuities of a shockwave, which are now second order and denoted as

$$[\partial_\mu \partial_\nu \phi_{\rho\sigma}]_\Sigma = \xi_\mu \xi_\nu \Phi_{\rho\sigma}. \quad (42)$$

From the field equation $\xi_{\mu\nu} = 0$ and its trace we learn that

$$[\mathcal{G}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \mathcal{G}_\rho{}^\rho]_\Sigma = \xi^2 \Phi_{\mu\nu} + \xi_\mu \xi_\nu \Phi - 2 \xi_{(\mu} \xi_{\nu)} \Phi = 0, \quad (43)$$

$$[\mathcal{G}_\mu{}^\mu]_\Sigma = -2\xi^2 \Phi + 2\xi_\mu \xi^\mu \Phi = 0, \quad (44)$$

($\Phi \equiv \Phi_\mu{}^\mu$). We now study the system for a causal timelike normal vector $\xi^2 = -1$. Note that since $\Phi_{\mu\nu} \neq 0$, we deduce that $V_\mu \equiv \xi_\mu \Phi_\mu \neq 0$ (otherwise $\xi^2 = 0$ and the model would be causal). So we now impose

$$\Phi_{\mu\nu} = -\xi_\mu \xi_\nu \xi \cdot V - 2\xi_{(\mu} V_{\nu)}, \quad \Phi = -\xi \cdot V \quad (45)$$

and study further constraints. In particular, the single divergence constraint gives

$$\xi^\mu [\partial_\mu D \cdot \mathcal{G}_\nu]_\Sigma = m^2 (V_\nu + \xi_\nu \xi \cdot V) + \frac{3ie}{2} (F_{\nu\rho} V^\rho - \xi^\rho F_{\rho\nu} \xi \cdot V + \xi_\nu \xi \cdot F \cdot V), \quad (46)$$

so that

$$\Pi_{\nu\rho} \left[m^2 \eta^{\rho\sigma} + \frac{3ie}{2} F^{\rho\sigma} \right] \Pi_{\sigma\tau} V^\tau = 0, \quad (47)$$

where the projector $\Pi_{\mu\nu} \equiv \eta_{\mu\nu} + \xi_\mu \xi_\nu$. It is sufficient to search for acausalities in constant background E/M fields, so with this restriction the double divergence constraint gives

$$\xi^\mu \xi^\nu [\partial_\mu \partial_\nu (D \cdot D \cdot \mathcal{G} + \frac{1}{2} m^2 \mathcal{G}_\rho{}^\rho)]_\Sigma = -\frac{3}{2} (m^4 - \frac{e^2}{2} F^2 + e^2 \xi \cdot T \cdot \xi) \xi \cdot V - 3e^2 \xi \cdot T \cdot V \quad (48)$$

Causality requires that the system of equations (47) and (48) have no non-zero solution for V_μ . In general (47) implies the vanishing of the components of V_μ orthogonal to ξ_μ and (48) in turn removes the parallel components. However if, regarded as a matrix equation in the orthogonal subspace, equation (47) fails to remove the orthogonal components of V_μ the model will be acausal. The determinant in this subspace vanishes whenever the (long-known) bound [5]

$$\bar{B}^2 = \left(\frac{2m^2}{3e}\right)^2. \quad (49)$$

is reached, Requiring that (48) be non-degenerate yields a different and weaker bound $\bar{B}^2 = (2m^4 + \bar{E}^2)/(3e^2)$. These differing bounds for the propagation of helicities zero and one are reminiscent of the behaviour found for $s = 2$ in cosmological backgrounds. Needless to say, these acausality and DoF problems are entirely independent of the formalism used to describe the initial free fields and to introduce couplings; these can only differ by non-minimal terms.

6. dS Supergravity?

Our considerations thus far have led us to the following picture of particles in (A)dS backgrounds. Partially massless fields, of either statistics, are always unitary in dS, while in AdS only strictly massless ones are. This behavior is understood by turning on cosmological constants of either sign and following their effects on the signs of lower helicity state norms. A sequence of unitary partially massless fields is only encountered when starting from Minkowski space ($\Lambda = 0$) and first turning on a positive cosmological constant. The bad news, however, is that partially massless fermions require tunings with negative m^2 as already noted in cosmological supergravity. This led to the rejection of dS supergravity as a consistent local QFT, a rejection bolstered later by the difficulties in defining string theory on dS backgrounds. Here I want to offer a few speculative remarks (every lecture should have some) revisiting this question.

Let us first present the reasons for rejection in terms of the present analysis, followed by such

mitigating circumstances as we can muster for keeping the possibility in play. For concreteness, we deal primarily here with $N = 1$ supergravity in four dimensions.

- Although (formally) locally supersymmetric actions exist, the mass parameter appearing in the term $m\sqrt{-g}\psi_\mu\gamma^{\mu\nu}\psi_\nu$ must be pure imaginary for lightcone propagation. Therefore, the action of dS supergravity does not obey a reality condition.
- The associated local supersymmetry transformations $\delta\psi_\mu = (D_\mu + \frac{1}{2}m\gamma_\mu)\varepsilon$, being complex, cannot preserve Majorana condition on fields. For $N = 2$ supersymmetry, a symplectic reality condition is possible, but the locally supersymmetric action is either still complex or the Maxwell field's kinetic term has tachyonic sign.
- Another way to see that there can be no real supercharges is that the $N = 1$ dS superalgebra ought be the $d = 5$, $N = 1$ superlorentz algebra, but there are no Majorana spinors in $d = 5$ Minkowski space.
- Because dS₄ has topology $S^3 \times \mathbb{R}$, gauge charges (being surface integrals) vanish since there are no spatial boundaries. This argument is of a different nature from the previous ones as it involves the global considerations which we have chosen to ignore.
- Finally, even for the $N = 2$ case, where a dS superalgebra exists, there are no positive mass, unitary representations. Unlike the praiseworthy AdS algebra, our maximal compact subalgebra has no $SO(2)$ factor whose generator could define a positive mass Hamiltonian. Again this is a global issue.

Let us now present the arguments in favor of dS supergravity:

- While the tuned dS supergravity mass $m^2 = -\Lambda/3$ is indeed negative, there are precedents for consistent theories with negative m^2 , for example scalars in AdS for

which a range of such values can be tolerated essentially because the lowest eigenvalue of the Laplacian has a positive offset there. Fermions mirror this behavior in dS.

- Despite an imaginary mass-like term in the action, at least the free field representations are unitary. For unitarity of spin 3/2 representations, the relevant quantity is $m^2 - 3\Lambda$, not m^2 alone. In addition the linearized equations of motion for physical spin 2, helicity ± 2 and its proposed spin 3/2, helicity $\pm 3/2$ superpartner degrees of freedom are identical.
- In dS space positivity of energy is possible only for localized excitations within the horizon. Only this region of dS possesses a timelike Killing vector. dS gravity is therefore stable against local excitations within the physical region. Although dS quantum gravity is problematic, nobody would reject cosmological Einstein gravity as an effective description of local physics within the physical region. This looser criterion is all we should require of a sensible dS supergravity.
- As stated, any local supersymmetry is purely formal in the absence of $d = 4 + 1$ Majorana spinors (even the fermionic field equation above is not consistent with a reality condition on ψ_μ). In any case, a bonafide $N = 1$ dS superalgebra with Majorana super charge, would imply a global positive energy theorem from $\{Q, Q\} \sim H$, which is already ruled out at the level of dS gravity. Instead, we could envisage relaxing the Majorana condition and allowing only a formal local supersymmetry. A direct Hamiltonian constraint analysis still shows that only helicities $\pm 3/2$ propagate. Also there are other examples where the Majorana condition must be dropped, but nonetheless a formal supersymmetry yields the desired Ward identities: continuation to Euclidean space is a familiar case.

The summary in favor of dS supergravity is then that the free field limit is unitary with time

evolution governed by the Hermitean generator $D = iM_{40}$ subject to a positive energy condition—within the physical region inside the Killing horizon. A formal supersymmetry, similar to that remaining when Euclideanizing supersymmetric Minkowski models, suffices to show that amplitudes obey supersymmetric Ward identities.

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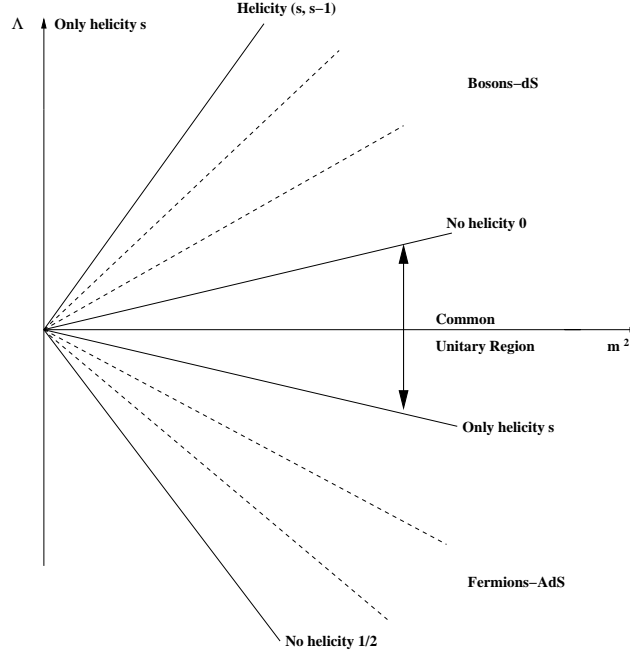


Figure 1. The top/bottom halves of the half-plane represent dS/AdS (and also bosons/fermions) respectively. The $m^2 = 0$ vertical is the familiar massless helicity $\pm s$ system, while the other lines in dS represent truncated (bosonic) multiplets of partial gauge invariance: the lowest has no helicity zero, the next no helicities $(0, \pm 1)$, etc. Apart from these discrete lines, bosonic unitarity is preserved only in the region below the lowest line, namely that including flat space (the horizontal) and all of AdS. In the AdS sector, it is the topmost line that represents the pure gauge helicity $\pm s$ fermion, while the whole region below it, including the partially massless lines, is non-unitary. Thus, for fermions, only the region above the top line, including the flat space horizontal and all of dS, is allowed. Hence the overlap between permitted regions straddles the $\Lambda = 0$ horizontal and shrinks down to it as the spins in the tower of spinning particles grow; only $\Lambda = 0$ is allowed for generic (m^2 not growing as s^2) infinite towers.